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PERFORMANCE COMPARISON OF MULTIPLE DISCRIMINANT ANALYSIS AND LOGIT MODELS IN BANKRUPTCY PREDICTION

ABSTRACT. In this study, the attention is dedicated to the development of bankruptcy prediction model in Slovak Republic. The presented paper focuses on the comparison of overall prediction performance of the two developed models. The first one is estimated via discriminant analysis, while the another is based on a logistic regression. The sample is made up of 236 firms operating in Slovakia, divided into two groups – failed and non-failed firms. The results of the study suggest that the model based on a logit function outperforms the classification accuracy of the discriminant model. The most significant predictors of impeding firms´ failure appear to be Net Income to Total Assets, Current Ratio and Current liabilities to Total Assets.

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Introduction

The outset of the global financial crisis is considered to be the year 2008, when the subprime crisis bubble in the United States bursted. Since then, we have observed to a much greater extent business cycle fluctuations resulting in numerous macroeconomic and microeconomic imbalances. Since companies are thought to be the economic subjects operating within the macroeconomic area, they are also affected by the abovementioned imbalances.

Because of the interconnectedness across national economies, Slovak companies also have been experiencing financial difficulties. Often, companies encounter a problem of unpaid bills, secondary insolvency, low law inforcement etc. These deficiences are easily transferred onto other companies. Such a tendency is also reffered to as contagion or knock-on effect. Likewise, it should be noted here that bankruptcy law in Slovakia often prefers the debtors´ interests over those of creditors. A clear evidence is provided by the well-known instances as Váhostav or Doprastav. Occasionally, there can be a situation when financially distressed company receive investment aid from the state or governmnet bodies. To avoid this type of situation, it is reasonable to have some early warning system capable of timely prediction of such situations.

Over the decades starting from the 1960s up to this point, there have been developed several early warning systems aiming at timely prediction of impeding companies´ financial difficulties. It is possible to group them into various categories. In the study by Taffler and Agarwal (2008), these models are divided into two groups: (i) accounting-based models; (ii) market-based models. An application of market-based models in Slovakia is restricted by the underdeveloped capital market. Therefore, in the process of bankruptcy prediction model development, application of accounting-based models is suggested. The use of accounting prediction methods has certain drawbacks, as indicated by the abovementioned authors, including the following: (i) accounting statements present past performance of a firm and may or may not be informative in predicting the future, (ii) conservatism and historical cost accounting mean that true asset values may be very different from the recorded book values, (iii) accounting numbers are subject to manipulation by management, and in addition, (iv) accounting statements are prepared on a going-concern basis. In spite of these limitations, it was confirmed that accounting-based models are not inferior in comparison to market-based models.

Even though application of bankruptcy prediction models is widely spread in Western advanced economies, it has become the fast growing research area in case of transitional economies, including countries of Central and Eastern Europe. Such a growing interest may result from several considerations, e.g. (i) financial institutions under Basel II guidelines are allowed to use their own internal ratings to assess the risk parameters of loan applicants; (ii) joining the EU structures, companies may take the opportunity to receive subsidy. In attempting to distunguish between well-established companies from distressed ones, failure prediction models are used; (iii) expanded activities of private equity financial groups. Specifically, its objective is to find potential company to invest in, merge or acquire. Prediction models may also serve as additonal tools in investors´ decision-making.

The focus in this study is on bankruptcy prediction model development based on two various statistical methods as applied to Slovakia. These includes both logistic regression function as well as multiple discriminant function under which we may find financial ratios best distinguishing among healthy and unhealthy companies. Thus, the main objective of this study is to compare the performance of the two proposed bankruptcy prediction models on a sample of selected firms operating in Slovak economic environment. Prediction models are estimated for the sample of Slovak healthy and unhealthy companies. On the basis of the existing literature (i. e., Charitou *et al.*, 2004; Hosmer *et al.*, 2013), we assume that because of limited statistical assumptions inherent in discriminant analysis model, the model based on logistic regression is inferior. The total number of firms identified as bankrupt was 118. Bankrupt firms were matched with nonbankrupt ones by asset size, industry. After univariate analysis of variable significance, the overall performance of the proposed models is evaluated. Our findings suggest that the model estimated through logistic regression is superior to that of multiple discriminant analysis.

The remainder of the paper is designed as follows: Section 1 provides a detailed survey of prior research and the related literature. Section 2 presents our methodology. Within this section, attention shifts from the methodology used to the description of the dataset. The empirical results are ummarized in Section 3. Section 4 includes the concluding remarks and suggests possible future research extensions.

1. Prior research and literature review

Kim (2011) discusses that over the periods there have been evolved two lines of research. The first group examines the incidence of failure aimed at the symptoms of failure and the another one is comparing the predcition accuracy of classification methods. This study is intended to contribute to the latter line of research by comparing the prediction accuracy of multiple discriminant analysis (MDA) to logistic regression.

The origin of the financial distress and bankruptcy examination went back to the sixthies of the last century in Beaver (1966) univariate analysis. Observations of the financial variables laid the groundwork for modern bankruptcy studies. Decision about companies´ financial sound based on the univariate analysis became the subject to the most criticism. The increasing number of critical voices gave rise the need for develop some multivariate techniques. In this regard, the pioneering study was presented by Altman (1968) using multiple discriminant analysis (MDA) to predict bankruptcy of manufacturing companies in United States. Altman (1968) pinpointed advantages of MDA over the traditional univariate analysis so that MDA is able to consider each of variable simultaneously as well as reduce their number.

However, as Eisenbeis (1977) reported, MDA has some statistical drawbacks making it difficult to apply. These drawbacks are being addressed by Premachandra *et al.* (2009) providing some limitations that mitigate the explanatory power of such models. The following are considered here: (i) propensity of equal variance-covariance matrices across the respective groups; (ii) the financial ratios entering in the model are multivariate normally distributed; (iii) the prior probability of the distress and costs of misclassifications are specified. Zavgren (1985) suggests that the generalizations and conclusions following from discriminant model characterized by violated assumptions are questionable.

Following the restrictions of MDA, the research focused on overcoming of restricted assumptions emerged as the prevailing prediction methods. In this context, there were developed conditional probability models, such as logistic regression (logit) or probit. The seminal work utilizing the logit methodology in bankruptcy prediction is that of Ohlson (1980). Research by Laitinen *et al.* (2005) suggests that the logit function is more sensible as it does not assume multivariate normality and equal covariance matrices as MDA does. Furthermore, MDA involves non-linear effects, enabling us to use logistical cumulative function in order to predict an impeding bankruptcy.

Although, there are novel techniques in predicting financial situation of companies, Kim (2011) stressed some benefits of MDA and logit. For instance, they can determine the importance of a variable, explain the results and there are many application software packages able to solve this problem. Additionally, a comprehensive study of Aziz and Dar (2006) recognizes that in the field of bankruptcy prediction, MDA and logistic regression have continued to be the frequently used solutions.

Subsequently, about 25 years ago, data mining techniques became to incorporate to bankruptcy prediction models. By their very nature, they surpass the drawbacks of traditional statistical techniques and functional form relating the dependent and independent variables. The group of data mining techniques comprises neural networks, case-based reasoning (CBR) and decision trees. The literature on using data mining techniques in bankruptcy prediction is reviewed in Olson *et al.* (2012). Referring to this study, decision trees are powerful classification algorithms that are becoming increasingly popular due to their intuitive explainability characteristics. Decision trees represents the fundamental tool in predicting bankruptcy in the study of Cardie (1993), Ahn and Kim (2009), Cho *et al.* (2010), Li *et al.* (2010).

The most prevalent bankruptcy prediction method using artificial intelligence is considered to be artificial neural networks (ANN). Following Kuumar and Ravi (2007), they are biologically inspired analytical techniques capable of modeling extremely complex non-linear functions. There are several variations of networks. The most common is the following: (i) multi-layer perceptron, (ii) self-organizing maps, (iii) probabilistic neural networks; (iv) learning vectors, and finally (v) Cascor. Among the first studies using neural networks in detecting financial distress were Odom and Sharda (1990), Wilson and Sharda (1994) and

Tam and Kiang (1992). The most recently studies includes, for example Kim and Kang (2010), du Jardin (2010) or Lee and Choi (2013).

Despite the outright explanatory power of neural network, Ahn and Kim (2009) noted that there are some difficulties in their using. These are arising from the fact that many parameters to be set by heuristics and therefore the model is exposed to overfitting. Finally, it leads to poor explanatory ability of the model. Thus, Watson (1997) suggests a case-based reasoning as an alternative to moderate the restrictions shown above. Yet, a study further provide the explanation, and that overfitting is not possible since it employs specific knowledges of experienced problems rather than their generalized patterns.

Supposedly, there is a tendency of exploiting more complex programming models based on expert, inteligent and mathematic systems. It should be demonstrated the following methods and their users: support vector machines (Cortes and Vapnik, 1995; Chen *et al.*, 2011; Li and Sun, 2011), genetic algorithms (Varetto, 1998; Daralos *et al.*, 2010), fuzzy set theory (Zarei *et al.*, 2011), rough set theory (Pawlak, 1982; Mosqueda, 2010) or integer programming (Glen, 1999; Xu and Papgeorgiou, 2009) or Bayesian probabilistic models (Sun and Shenoy, 2007). This study, by developing of both MDA and logit models, aims to empirically explore their prediction accuracy on the sample of Slovak companies. The contribution of this study is to find which of the devised model has the more explanatory power to ahead predict impeding financial distress of companies. There is a research gap in this area in the Slovak republic and therefore, we hope this study forces the research attempts in bankruptcy prediction.

2. Methodological aspects

This section describes theoretical basis of models employed, data used, sample design and variable selection procedure. To estimate bankruptcy prediction model, presented paper utilize two sort of statistical methods, including multiple discriminant analysis and logit regression. The method choice arises from its extensive application in bankruptcy prediction literature up to this point. Tinoco and Wilson (2003) pointed that, even though the restricetd statistical assumptions, considered methods have been continued in its utilization. One of the most relevant advantage of these methods is its ability to easy interpret accomplished conclusions. Furthermore, there is an extensive software packages enabling us to user friendly interface. On the other hand, we have to admit some restrictions associated with using these methods. Admittedly, we should refer to a statistical assumptions, such as multivariate normality, homoscedasticity of data etc. in the cas eof multiple discriminant analysis. Since, logistic regression method do not require statistical assumption meeting, we decided to compare the prediction accuracy of methods in bankruptcy prediction. Simply stated, we would like to hypothesize whether non-linearity inherent in logistic regression gives incremental information when compared to multiple discriminant analysis.

2.1. Multiple discriminant analysis

According to Tabachnick and Fidell (2001), the goal of discriminant analysis is to predict group membership from a set of predictors. Back *et al.* (1996) explicitly address this issue and emphasized that discriminant analysis seeks to find the linear combination of two or more predictors capable of discriminate at best among a pre-determined groups of failing or non-failing companies. It may be attained by maximizing the between group variance relative to the within group variance. This relationship is given by the Fisher´s criterion function and takes the following form:

$$J(w) = \frac{w^T(\sum_i (x_i - \mu)^T (x_i - \mu))w}{w^T(\sum_c \sum_{i \epsilon c} (x_i - \mu_c)^T (x_i - \mu_c))w}, \qquad (1)$$

where, *w* denotes the projection matrix that maximizes the ratio of the determinants of between group variance to the determinant of the within group variance; x_i presents the values of the samples; μ is sample mean; w^T refers to transposed projection matrix; summation over *c* means summing within class, μ is group mean for class *c*.

Discriminant method estimates a discriminant function with coefficient vector *A* (a_1, a_2, ..., a_n). In this respect, we follow the study of Dimitras *et al.* (1996) that indicates the linear combination of the independent variables in this manner:

$$Z_i = a_0 + a_1 x_{i1} + a_2 x_{i2} + a_3 x_{i3} + \cdots + a_n x_{in}, \qquad (2)$$

where Z_i is the discriminant score for i-th company and $x_{i1} x_{i2}, \ldots, x_{in}$ are the *n* variables for i-th company.

The study by Altman (1968), was the first one using the multiple discriminant analysis in corporate failure prediction. A discriminant function proposed by him is termed as Altman´s Z-score. This study has become the most cited paper regarding bankruptcy prediction research resulting in some contradictory questions. The most common discussed question relates to violation of underlying assumptions required by the model application. These issues are analyzed in the numerous studies, specifically Eisenbeis (1977) or Tabachnick and Fidell (2001). These studies give some indication and reports limitations including: (i) unequal sample sizes, missing data and power, (ii) multivariate normality, (iii) absence of outliers, (iv) homogeneity of variance-covariance matrices, (v) linearity, (vi) absence of multicollinearity and singularity.

Above stated restrictive limitations will be tested. Testing of discriminant model assumptions may indicates whether the discriminant model is usable. Apparently, violation of normality assumption seems to be the major concern in using financial ratios to predict financial distress of companies. In this vein, there is a need for testing the normality of financial ratios on the basis of univariate test. For this purposes, Wilk´s Shapiro test will be employed. The nature of this test strings from the null hypothesis that a sample come from normally distributed population. Related test statistics is as follows:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n} (x_i - \overline{x})^2}, \qquad (3)$$

Discriminant analysis is to a great extent sensitive to occurence of outliers. To find and eliminate significant univariate outliers. Hampel´s test will be conducted. Lasisi and Shangodouin (2014) outlined that Hampel had proposed an identifier exploiting the median to estimate data location and median absolute deviation to estimate the standard deviation. The process of identifying Hampel test is the following:
(i) compute the median $\tilde{x}$,
(ii) compute the MAD that presents an outlier-resistant alternative to the standard-deviation, in that manner

$$MAD = median\{|x_i - \tilde{x}|\}, \qquad (4)$$

(iii) Hampel's method identifies x_i as outlier when $|x_i - \tilde{x}| > 5.2\ MAD$, (5) In order to find whether covariance matrices are equal, Box´s M test will be used. I tis considered as Bartlett´s test in the multivariate expression. To undertake ths test, the assumption of normality has to be met. We test the hypothesis that the population covariance matrices are all equal:

$$H_0: \Sigma_1 = \Sigma_2 = \cdots \Sigma_m, \qquad (6)$$

where m is the number of independent populations. Now, assume that S_1, ..., S_m presents sample covariance matrices from the m populations, df is degrees of freedom and every S_j is of n_j independent observations comprising of $k\,x\,1$ column vector. Now we can define covariance matrix of sample:

$$S = \frac{1}{n-m}\sum_{j=1}^{m}(n_j - 1)S_j, \qquad (7)$$

$$M = (n-m)ln|S| - \sum_{j=1}^{m}(n_j - 1)ln|S_j|, \qquad (8)$$

$$c = \frac{2k^2+3k-1}{6(k+1)(m-1)}\left(\sum_{j=1}^{m}\frac{1}{n_j-1} - \frac{1}{n-m}\right), \qquad (9)$$

The test statistic of Box´s M test is then:

$$M(1-c) \sim \chi^2\,(df), \qquad (10)$$

Having underlying assumption verified, MDA calculates the discriminant coefficients and discriminant score for each of the included company accordingly Eq.2. An additional procedure select appropriate cut off score which will preserve the essence of Fisher´s criterion function and maximizes the ratio of the between-group variance to the within-group variance. Chung *et al.* (2008) suggest that, by using the Z score and cut off score, a company is classified into failed or non-failed categories.

2.2. Logistic regression

As stated by Hair *et al.* (2006), logistic regression is the appropriate statistical method when the dependent variable is a categorical variable, whereas the independent variables ate nonmetric or metric variables. In addition, they provide that logistic regression is commonly used for two reasons: (i) logistic regression is not required meeting statistical assumptions; (ii) logistic regression has straightforward statistical tests and includes non-linear effects. As discussed in Kolari *et al.* (2002), if the assumptions of discriminant analysis hold, logistic regression is equivalent to MDA.

Logit model is based on a cumulative logistic function. As a result of this, we obtain the probability of a company belonging to one of the a priori determined gropus, in view of the financial features of the company. The probability of a company to go bankrupt (PL) is computed employing the cumulative logistic function:

$$P_{L_a} = \frac{1}{1+e^{-(Z_{L_a})}}, \qquad (11)$$

where

$$Z_{L_a} = \beta_1 x_{1a} + \beta_2 x_{2a} + \cdots + \beta_n x_{na}, \qquad (12)$$

Following the Hosmer and Lemeshow (2013), one can define the logit as:

$$logit\left(P_{L_a}\right) = ln\left(\frac{P_{L_a}}{1-P_{L_a}}\right) = f(x,\beta) = \beta^T x \qquad (13)$$

whereβ= (β_0, β_1, β_2, ... β_k) is the vector of the coefficients and $\beta^T x$ is the transposed vector. The relationship $\frac{P_{L_a}}{1-P_{L_a}}$ is referred to as odds ratio and ln of this relationship denotes logit transformation.

The unknown coefficients B_i are estimated from the data using the maximum log-likelihood method:

$$l(\beta) = \sum_{i=1}^{N}\{y_i \ln\left(P_{L_a}\right) + (1 - y_i)\ln(1 - P_{L_a})\}, \qquad (14)$$

On the basis of adopted probability, using a cut off score, a company is classified as failing or non-failing. The emphasis is placed on the minimizing Type I error (failing company classified as non-failing) and Type II error (non-failing company classified as failing).

In the context of bankruptcy predition, Ohlson (1980) was the first who used logistic regression to model non-linear relationship in his study. This methodology has been also used in the variety of studies, recalling Zavgren (1985), Keasey and Watson (1991), Premachandra (2011), Chen (2011).

2.3. Data and sample selection

While proposed discriminant and logit model is based on accounting information, the principal data are collected from the annual financial reports of companies. The sample is composed of two groups of companies. The first one is formed by the financial healthy companies and the other consists of unhealthy companies. Over time, there have been established the variety of definitions of the terms insolvency, bankruptcy, failure, financial distress, financial difficulties, financial soundness, financial health etc. Bellovary *et al.* (2007) deduced that such a ambiguity in definitions makes it difficult mutual comparability of models. It presents the crucial limitation in companies´ bankruptcy prediction. Furthermore, Tinoco and Wilson (2013) find that legal date of failure does not need to be real date of the financial difficulties outset. These findings were subsequently confirmed by Bauer and Agarwal (2014), when according to them, there is a considerable time gap (up to three years, or 1.17 years on average) between the period of financial distress outset and the legal date of failure.

In this study, the legal definition of failure is utilised. The reasons for considering this type of definition is emphasized by Charitou *et al.* (2004): (i) it provides objective criterion, enabling us to easily classify the set of companies, (ii) failure date is objective determined. In the previous studies, there have been recorded instances that do not distinguish among the variety of definitions. Similar to Altman *et al.* (2014) procedure, we consider terms liquidation, restructuring and failure as identical. Thus, the company is included in the failing group of sample if it satisfies one out of the conditions: (i) the company files a bankruptcy petition, (ii) the company ceases operation or is in liquidation, and finally (iii) the company is allowed to initiate the restructuring process. The failing subsample is comprised of 118 companies meeting one out of the above prescribed suppositions.

Subsequently, the group of healthy companies is composed to setting the model. To this end, paired-sample design was used. Once we have subsample of failing companies, the subsample of non-failing companies is designed meeting some criteria. Each failing company is paired with those of non-failing based on the asset size and industry correspondence. The approach of matched samples were also used by Charitou *et al.* (2004) or Karas and Režňáková (2012). Very nature of the paired-sample design makes sure that the sample of unhealthy companies is also made up by the 118 of them. Hence, the final sample consists of 236 companies. It is of important to note that non-failing companies are retrieved from the same year as their failing counterpart.

The sample of the study covers the period of the year 2014 and is composed of 118 of failing as well 118 of non-failing companies. As to a data set, accounting information were adopted from databse ORBIS of Bureau van Dijk. The data were collected as follows: if company failed in 2014, accounting data from the 2013 are considered. In the same way, accounting data from the 2014 for non-failing company are taken into consideration. The corresponding data regarding the financial situation of company (failed or non-failed) are gathered from CRIBIS database and Obchodný vestník SR.

The variable selection procedure follows the approach suggested by Mihalovič (2015). An approach used in this study includes the variables significant in previous studies, for example Psillaki *et al.* (2009) or Laitinen and Lukason (2014). For the purposes of this study, we follow this convention. The initial set of variables under consideration is drawn from the 18 variables, from which the final set of variables based on pairwise testing is developed.

2.4. Evaluation methods

The basis for companies´ classification is formed by the finding of optimal cut off score. Canbas *et al.* (2005) discusses that a company from a priori group is classified as failing or non-failing according to whether its predicted probability falls below or above a cut off score. The right selection of cut off scorre determines the classification results. It is the traditional tradeoff issue between the probability of Type I and Type II errors. A detailed description of error rates calculation is provided by Chen (2011), who put forward the following procedure:

(i) the probability of failure for each company is calculated;

(ii) reclassification of each company according in two groups to comparison of calculated probability of failure to a cut off score (probability);

(iii) if an estimated probability of failure for failing company is below the cut off score, this company is misclassified by the model;

(iv) if an estimated probability of failure for non-failing company is above the cut off score, this company is misclasdified by the model;

(v) the error rates in every group are calculated by dividing the number of misclassified companies by the total number of companies in the group. Corresponding error rates are referred to as Type I and Type II error.

To acquire the error rates, optimal cut off score has to be computed.Hair *et al.* (2006) defines the cut off scores as a dividing point used to classify observations into groups based on their function score. The calculation of a cut off score between any two groups is based on the two group centroids and the relative size of the two group. If the group sizes are equal (prior probabilities are 0.5), the optimal cutting score takes the following form:

$$CS_{opt} = \frac{N_A Z_B + N_B Z_A}{N_A + N_B}, \qquad (15)$$

where CS_{opt} is optimal cut off score between groups A and B, N_A, N_B are sizes of group A and B, Z_A, Z_B are centroids for both groups A and B, respectively.

The objective of this procedure is to minimizes the sum of the Type I and Type II errors, putting equal weight of 50% on Type I and Type II errors (Bryant, 1997). Although, as a practical matter, the cost of Type I and II are not the same, we follow the convention involved in the studies Bryant (1997) or Ohlson (1980) assuming that weights of error rates are equal.

Ultimately, the overall fit of the model is performed to compare actual and predicted companies´ membership into corresponding groups. The meaningful output of this procedure is the hit ratio representing the percentage of objects correctly classified by the model. Essentially, it is the number of objects in the diagonal of the classification matrix divided by the total number of objects. Lastly, the significance of the classification accuracy is conducted through *t*-test:

$$t = \frac{p-.5}{\sqrt{\frac{.5(1.0-.5)}{N}}}, \qquad (16)$$

where p is the proportion correctly classified, N – sample size.

2.4.1. Confusion matrix

The first one assessment tool is that of confusion matrix, also referred to as contingency table. Such a table compares the number of correct and incorrect firms´ classification based on actual and predicted values.

Table 1. Confusion matrix

		Predictive value		
		0 (non-bankruptcy)	1 (bankruptcy)	
Actual value	0 (non-bankruptcy)	*A (TP)*	*B (FP)*	TP+FP
	1 (bankruptcy)	*C (FN)*	*D (TN)*	FN+TN
		TP+FN	FP+TN	total

Source: own research.

Table 1 shows the breakdown of the number of predictive and actual values for bankrupt and nonbankrupt firms. *A* denotes the number of firms predicted by the model as nonbankrupt as well as actually nonbankrupt. *B* indicates the number of actually bankrupt firms also predicted by the model as bankrupt. *C* indicates the number of actually bankrupt firms predicted by model as nonbankrupt. Lastly, *D* indicates the number of bankrupt firms, that is confirmed in this wayalso by the model prediction.

The cells in the table indicate the percentage of true positives (*TP*), false positives (*FP*), false negatives (*FN*) and true negatives (*TN*).

2.4.2. Receiver operating characteristic (ROC)

A more complete description of classification accuracy is given by the area under the ROC curve. This curve, as discussed in Hosmer and Lemeshow (2013), originates from signal detection theory and shows how the receiver operates the existence of signal in the presence of noise. It plots the probability of detecting true signal (sensitivity) and false signal (1-specificity) for an entire range of possible cut-off points.

The area under the ROC curve, which ranges from zero to one, provides a measure of the model´s ability to discriminate between those subjects who experience the outcome of interest versus those who do not. It is constructed by varying cut-off points mapping estimated probabilities of default on class prediction. Reisz and Perlich (2007) state that for every cut-off point, the ROC demonstrates the true positive rate (D in confusion matrix above) on the y- axis as a function of the corresponding false positive rate (B in the confusion matrix) on the x-axis.

The way of ROC parameters calculation is given by the formula:

$$FP\ rate = \frac{FP}{FN+TN}, \qquad (17)$$

$$TP\ rate = \frac{TP}{TP+FN}, \qquad (18)$$

$$accuracy = \frac{TP+TN}{P+N}, \qquad (19)$$

In bankruptcy prediction context, as indicatede by Fawcett (2006), ROC curve displays the Type II error (FP rate) on the x-axis against the corresponding Type I error (TP rate) on the y-axis. From the lender perspective, Type II error presents nonbankrupt firms that must be denied credit in order to avoid granting a loan to a specific percentage of deafulting firms (Type I error) when applying a specific bankruptcy prediction model.

The classification accuracy of the model is assessed through area under the ROC curve. Agarwal and Taffler (2007) proposed the Wilcoxon statistic as a basis for estimation of area under the ROC curve.

3. Empirical results

Throughout the research process, two models were estimated. One of them is estimated based on discriminant function and the another uses logistic regression. In the first place, results are assessed separately. Afterwards, the classification accuracy of corresponding models is assessed.

As previously indicated, estimation of discriminant function requires meeting some underlying assumptions. Our findings concerning normality of financial ratios are consistent with the results of Piotroski (2000), since observed financial ratios do not follow the normal distribution. The only one financial ratio, having the properties of normal distribution involves Working capital/ Total assets. The crucial matter, in this sense, is whether the violation of normal distribution is due to outliers or skewness. If the non-normality is the result of skewed data, Tabachnick and Fidell (1996) argue that violation of normality assumption is not so profound. The multicollinearity was evaluated by Pearson´s correlation matrix. It was found, that ratios including Working Capital/Total Assets and Current liabilities/Total Assets show the high degree of negative correlation (-0.9525). Following Cochran (1964), such a high degree of negative correlation is more helpful in adding new information to the discriminant function as high degree of positive correlation.

3.1. Discriminant function

Firstly, it was required to confirm the meeting of restricted assumptions. After that, the canonical discriminant function was estimated in the following form:

$$D(f) = -0.507x_1 - 0.263x_2 + 0.271x_3 + 0.235x_4 + 0.526x_5 \qquad (20)$$

where x_1=Net income/Total Assets, x_2=Current ratio, x_3=Current liabilities/Total Assets, x_4=Working Capital/Total Assets, x_5=Current Assets/Total Assets.

Since the discriminant function has multivariate properties, it is appropriate to perform multivariate test of explanatory power of financial ratios.

Table 2. Results of discriminant function

Variable	Wilk´s lambda	F-statistic	p-value
NI/Total Assets	0.9689	7.5084	0.007*
Current Ratio	0.9463	13.2554	0.000*
Current liab./TA	0.9766	5.5977	0.019*
Working capital/TA	0.9868	3.1001	0.079
Current Assets/TA	0.9990	0.2275	0.634

Function	Eigenvalue	Canonical R^2	Wilk´s lambda	Chi-square	df	p-value
1	0.10007	0.0909	0.372	20.181	4	0.000*

Structure matrix	
Independent variable	Discriminant correlation
NI/Total Assets	-0.5846
Current Ratio	-0.7677
Current liab./TA	0.5068
Working capital/TA	-0.3791
Current Assets/TA	-0.1033

Source: own research.

Table 2 groups the empirical results achieved by the discriminant function estimation. From the table, it is obvious that three ratios including Net Income/Total Assets, Current ratio, Current Liabilities/Total Assets best separate between the groups of bankrupt and non-bankrupt firms. From the univariate view of variables significance, the only ratio Current Assets/Total Assets is not significant in distinguishing between healthy and unhealthy firm.

The results of overall Wilk´ s lambda test indicate that the independent variables are not equal between the groups of dependent variable (p-value 0.000). Thus, one can conclude that canonical discriminant function well separate between two heterogenous groups of firms. Also, it was noticed that the 37.2 percent of variance in discriminant scores is not explained by group differences. Although, Wilk´s lambda indicates well-performed model, in the future research it is required to focuses on the stepwise estimation process keeping only variables that separate between groups of companies on the univariate basis.

The last part of table lists the correlations between independent variable and dependent variable according to whether firm is classified as bankrupt or not. The results of structure matrix implies that each of observed independent variable contribute to dependent variable explanation in the significant way. The highest degree of relationship is indicated by Current ratio in the negative sense. It stands for that the lower level of Current ratio, the higher probability of firm´s bankruptcy. The similar negative relationship is also recorded in the case of Net Income/Total Assets, Working capital/Total assets and Current assets/Total assets.

3.2. Logistic regression

In addition to dicriminant function, the logit regression model was estimated. Recalling that the coefficients of the function were estimated by the maximum likelihood function, the yielding logistic function is:

Table 3. Estimated logistic regression function

Coefficients	Estimate	Standard Error	z-value	p-value
Intercept	-1.01044	0.61061	-1.655	0.09797 *
NI/Total Assets	-0.73287	0.83319	-0.880	0.37908
Curremt Ratio	-0.08631	0.13744	-0.628	0.53000
Current liab./TA	1.05539	0.61579	1.714	0.08655*
Working capital/TA	-2.09519	0.79100	-2.649	0.00808 ***
Current Assets/TA	0.54097	0.85458	0.633	0.52672
Significant codes: *** (0.001); ** (0.01); *(0.1)				
Null deviance	Residual deviance	Akaike information criterion		
163.28 on 117 degrees of freedom	133.06 on 112 degrees of freedom	145.06		
Significance of deviance differences				p-value: 0.0851

Source: own research.

The effect of individiual logistic regressors on dependent variable is performed through Wald´s Z test statistic. The fitting regression model shows that based on Wald´s Z-statistic, Current liabilities/Total Assets and Working capital/Total assets are significant predictors of firm´s bankruptcy given logistic regression. The resulting logistic function takes the following form:

$$\hat{p}(x) = \frac{e^{-(1.01044-0.73287x_1-0.08631x_2......)}}{1+e^{-(1.01044-0.73287x_1-0.08631x_2......)}}, \tag{21}$$

The overall performance of the logit model is evaluated by comparing the null and residual deviance. From the *Table 3*, it is obvious that including the independent variables decreased the deviance to 133.06 points. The test of statistical significance of deviances differences indicates that null hypothesis is not rejected and therefore the fitted values are not significantly different from observed values. However, the logit model in this form is not valid, since the overall significance of deviance differences is not sufficient (p-value is 0.0851).

Finally, analysis of deviances (ANOVA) is performed as a proxy for likelihood ratio test. It tests the null hypothesis that adding the variables into model do not convey an additional information.

Table 4. Analysis of deviances

Coefficients	Df	Deviance Residuals	Df	Residual deviance	p-value(χ^2)
NULL			117	163.28	
NI/Total Assets	1	10.0537	116	153.22	0.001520 **
Curremt Ratio	1	5.3744	115	147.85	0.020434 *
Current liab./TA	1	6.9092	114	140.94	0.008575 **
Working capital/TA	1	7.4807	113	133.46	0.006236 **
Current Assets/TA	1	0.4035	112	133.06	0.525301
Significant codes: 0(***); 0.001(**); 0.01(*)					

Source: own research.

Analysing the deviances, we can see the decrease in deviance when adding each independent variable one at a time. Adding Net Income/Total Assets, Current Ratio, Current liabilities/Total Assets and Working capital/Total Assets significantly reduces the residual deviance. A large p-value regarding variable Current Assets/Total Assets suggests that the model without that variable explains more or the same amount of variation.

Results of prediction for logistic regression model along with the cut-off point finding are displayed in *Table 5*:

Table 5. Accuracy of logistic regression and optimal cut-off

	Column heading = probability cut-offs								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Training data									
Failed firms	0.125	0.268	0.357	0.500	0.643	0.804	0.911	0.946	0.982
Non-failed firms	0.984	0.984	0.952	0.855	0.790	0.677	0.403	0.194	0.032
Totals	0.576	0.644	0.694	0.684	0.720	0.737	0.644	0.551	0.483
Type I error	0.983	0.983	0.952	0.855	0.790	0.677	0.403	0.194	0.033
Type II error	0.875	0.732	0.643	0.500	0.357	0.214	0.089	0.054	0.019
Total error rate	0.424	0.356	0.331	0.314	0.279	0.263	0.356	0.449	0.517
Testing data									
Failed firms	0.129	0.177	0.209	0.306	0.403	0.613	0.839	0.919	0.952
Non-failed firms	0.964	0.911	0.875	0.839	0.768	0.661	0.518	0.250	0.054
Totals	0.525	0,525	0.525	0.559	0.576	0.636	0.686	0.616	0.525
Type I error	0.871	0.822	0.790	0.758	0.694	0.597	0.467	0.226	0.048
Type II error	0.964	0.911	0.875	0.768	0.661	0.429	0.196	0.089	0.054
Total error rate	0.475	0.475	0.475	0.441	0.424	0.364	0.314	0.398	0.475

Source: own research.

Once the logit model is estimated, it is required to find optimal cut-off point to properly classify the firm´s bankrupt or not. As earlier indicated, the optimal cut-off point is that characterizing by the minimizing of Total error rate. *Table 5* reveals that based on training dataset, the optimal cut-off point appears to be 0.6 with the total error rate at the level of 26.3%. In association with the optimal cut-off point, the accuracy rate (73,7%) give the evidence of proper choice of cut-off point. As you can see, the choice of optimal cut-off point is typical trade-off problem, since accuracy of failed firms move in opoosite way in comparison with non-failed firms.

3.3. Classification accuracy

In order to evaluate the overall performance of the estimated model (discriminant model and logit model), classification (confusion) matrix and area under the ROC curve was employed. The results of classification accuracy using confusion matrix are provided by the *Table 6*.

It is worthwile to emphasize that overall classification accuracy is assessed in view of testing data. Regarding the overall prediction accuracy of discriminant function, it is of interest to note that the performance of testing data (61.86 percent) surpasses those of training data (64.41 percent). In comparing the models between the each other, logit model presents higher predictive performance both in the testing data and in the training data. In the case of the total accuracy of training data, logit model overcomes discriminant function for 11.87

percent, while in testing data for 4.23 percent. One would conclude that based on testing data, the difference in classification accuracy is not so evident.

Table 6. Confusion matrix of discriminant and logit estimation results

Classification Results (discriminant function)				
		Predicted Group Membership		
	bankrupt or not	bankrupt	non-bankrupt	total
Training data	Bankrupt	35	26	61
	non-bankrupt	19	38	57
Total accuracy				61.86%
Test data	Bankrupt	26	31	57
	non-bankrupt	11	50	61
Total accuracy				64.41%
Classification Results (logit function)				
		Predicted Group membership		
	bankrupt or not	bankrupt	Non-bankrupt	total
Training data	Bankrupt	42	19	61
	non-bankrupt	12	45	57
Total accuracy				73.73%
Test data	Bankrupt	29	26	55
	non-bankrupt	11	52	63
Total accuracy				68.64%

Source: own research.

Consequently, ROC curve were designed in order to include the additional measure of model performance. The graphical illustration of ROC curve is recorded on the following Figures.

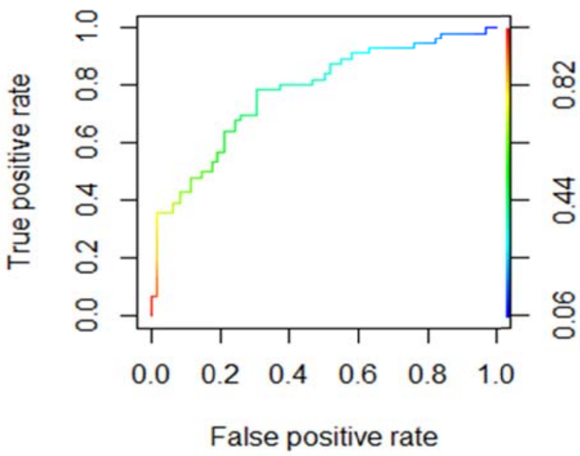

Figure 1. ROC curve for logit function
Source: own research.

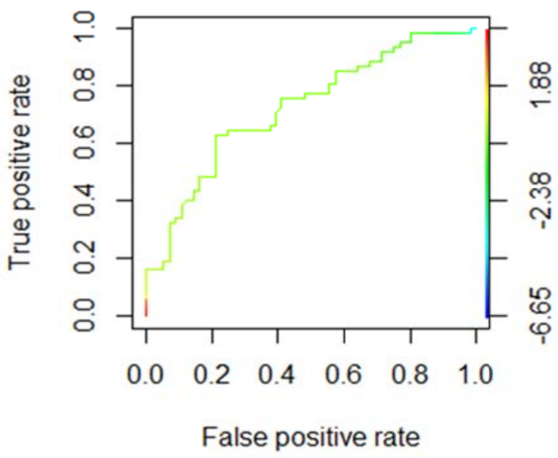

Figure 2. ROC curve for discriminant function
Source: own research.

Figures 1 and *2* demonstrate that area under the ROC curve for logistic regression is greater than fot that of discriminant function. The evidence of this assertion is provided by the following table:

Table 7. Area under the ROC curve ratio and accuracy ratios

Model	Area under curve	Accuracy Ratio=2*(AUC-0.5)
Logit	0.772	0.544
Discriminant analysis	0.723	0.446

Source: own research.

Based on the *Table 5* results, we may observe that the lowest degree of total error rate for logistic regression training data is achieved at the cut-off point 0.6. Specifically, it corresponds to the value of 0.263. In other words, logistic regression model is able to correctly classify 73.6 percent of cases. Unlike, for testing data, the minimal total error rate was obtained at the level of cut-off point 0.7. The overall prediction accuracy of logistic regression conducted on a testing data was 68.6 percent.

However, above mentioned results indicates only correct and non-correct results for one cut-off point (for which the total error rates is the lowest). Apart from this, figure Receiveng operating characteristic (ROC) represents a metric of correct and non correct classification accuracy for many different cut-off points. In this context, Area under curve (AUC) is a measure telling us what is the probability when we ramdomly draw the observation, that model correctly classify obseravtion to the actual group. So, in the case of logit model, there is 77.2 percent probability of correctly classification firm into the actual group. It is inferior in compare with discriminant analysis model (72.3 percent). However, observed difference is not so conclusive.

Regarding true positive and true negative rate, it is common trade-off dispute, since the best results are achieved when true positive rate upcomes to 1 and false positive rate to 0. In this case, AUC area will be 1 indicating the model is perfect in classifying objects into the pre-determined groups. The best results are achieved when the true positive rate is 0.677, while false positive rate is 0.214. It stands for that having such a model using in the creditworthiness assessment of applicants, based on our model, in the 21.4 percent of cases we extend credit to loan applicants that fail in the future. This deficiency represents some type of the model imperfection.

Conclusions

The presented paper broaded the bankruptcy prediction model discussion. Within the bankruptcy prediction, in Slovakia there has not been developed a generally accepted model. Thus, this paper has attempted to span this drawback. To this end, two prediction models based on discriminant analysis and logistic regression were estimated.

Both bankruptcy prediction models were estimated using accounting-based data on matched sample of Slovak healthy and unhealthy groups of firms. The sample covers the period of year 2014 and the models estimated are evaluated by means of confusion matrix and receiver operating characteristics. Variables of estimated models were adopted by univariate analysis of predictive power of variables. In addition, variables in the multiple discriminant model had to be adjusted due to the non-normality, multicollinearity and outliers presence. After doing that, there was estimated MDA model including five accounting ratios.

From that variable set, models estimated suggests that ratio Current assets/total assets is not significant, meaning that it does not distinguish well between healthy and unhealthy firms. The remaining four variables were significant. Using structure matrix, we may consider Current ratio as the best separator in negative sense. Thus, the more Current ratio value, the lower probability of firm´s failure. Concerning the other ratios, Net income/Total assets and Current liabilities/Total assets contribute to prediction power of model. Only one ratio including Current liabilities/total assets have positive discriminant correlation, following that the higher magnitude of current liabilities with respect to total assets, the higher probability of firm´s bankruptcy. In spite of Current assets/Total assets insignificance, overall prediction performance of model is sufficient arguing by Wilk´s lambda parameter. However, results obtained by MDA estimaton can not be overstated, since the canonical coefficient of determination is too low.

In summary, MDA is not recommended method for bankruptcy prediction because of: (i) model does not explain the adequate proportion of variability (low canonical coefficient of determination; (ii) some statistical assumptions are violated – multicollinearity, presence of outliers, non-normality of ratios values; (iii) note ach of variables included in model is significant. Within the given constraints, it is possible to consider stepwise variable selection that allow us to comprise only significant variables.

In terms of logit model estimation, we have to admit its uselessness. Such an assertion is evidenced by the overall significance of deviance differences (p-value-0.0851). It indicates that fitted values are not significantly different from observed values. It may also lead us to conclusion that model in its proposed version are not applied in the Slovak business environment. This is might due to the fact that there were inappropriate variables selected. Another reason we can find in the limited abilities of statistical modeling techniques.

To overcome observed deficiencies, it is appropriate use other prediction techniques including artificial intelligence expert systems. These techniques do not require the statistical assumption fulfillment. Likewise, previous literature on bankruptcy prediction models have demonstrated the higher performance of data mining techniques relative to statistical techniques. Inferences drawn from this study indicates that there are opportunities and blank spaces in the area of bankruptcy prediction. The futures research is intended to be find the possibilities to secure the overall model perfomance or develop novel model approaches.

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References

Agarwal, V., Taffler, R. (2008), Comparing the performance of market-based and accounting-based bankruptcy prediciton models, *Journal of Banking and Finance*, Vol. 32, No. 8, pp. 1541-1551.

Ahn, H., Kim, K. (2009), Bankruptcy prediction modeling with hybrid case-based reasoning and genetic algorithms approach, *Applied Soft Computing*, 9.2, pp. 599-607.

Altman, E. I. (1968), Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *The journal of finance*, Vol. 23, No.4, pp. 589-609.

Altman, E. I., et al. (2014), *Distressed Firm and Bankruptcy Prediction in an International Context: A Review and Empirical Analysis of Altman's Z-Score Model*, available at SSRN 2536340.

Aziz, M. A., Dar, H. A. (2006), Predicting corporate bankruptcy: Where we stand? *Corporate Governance*, Vol. 6, No. 1, pp. 18-33.

Back, B., Laitinen, T., Sere, K. (1996), Neural networks and genetic algorithms for bankruptcy predictions, *Expert Systems with Applications*, 11.4, pp. 407-413.

Bauer, J., Agarwal, V. (2014), Are hazard models superior to traditional bankruptcy prediction approaches? A comprehensive test, *Journal of Banking & Finance*, 40, pp. 432-442.

Beaver, W. H. (1966), Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, *The journal of Accounting Research*, Vol. 4, pp. 71-111, ISSN 0021-8456.

Bellovary, J. L., Giacomino, D. E., Akers, M. D. (2007), A review of bankruptcy prediction studies: 1930 to present, *Journal of Financial Education*, Vol. 33, No. 4, pp. 3-41.

Bryant, S. M. (1997), A case-based reasoning approach to bankruptcy prediction modeling, *Intelligent Systems in Accounting, Finance and Management*, 6.3, pp. 195-214.

Cortes, C., Vapnik, V. (1995), Support-vector networks, *Machine learning*, 20.3, pp. 273-297.

Dimitras, A. I., Zanakis, S. H., Zopounidis, C. (1996), A survey of business failures with na emphasis on prediction methods and industrial applications, *European Journal of Operational Research*, Vol. 90, No. 3, pp. 487-513.

Eisenbeis, R. A. (1977), Pitfalls in the application of discriminant analysis in business, finance, and economics, *The Journal of Finance*, 32.3, pp. 875-900.

Hair, J. F., et al. (2006), *Multivariate data analysis*, Upper Saddle River, NJ: Pearson Prentice Hall.

Hosmer, J. R, Lemeshow, S., Sturdivant, R. X. (2013), *Applied logistic regression*, John Wiley & Sons.

Charitou, A., Neophytou, E., Charalambous, C. (2004), Predicting corporate failure: Empirical evidence for the UK, *European Accounting Review*, Vol. 13, No. 3, pp. 465-497.

Jardin, P. (2009), Bankruptcy prediction models: How to choose teh most relevant variables? *Bankers, Markets & Investors*, No. 98, pp. 39-46.

Karas, M., Režňáková, M. (2012), Financial Ratios as Bankruptcy Predictors: The Czech Republic Case, *Proceedings of the 1st WSEAS International Conference on Finance, Accounting and Auditing (FAA'12)*, pp. 86-91.

Keasey, K., Watson, R. (1991), Financial distress prediction models: A review of their usefulness, *British Journal of Management*, Vol. 2, pp. 89-102.

Kim, M. J., Kang, D. K. (2010), Ensemble with neural networks for bankruptcy prediction, *Expert Systems with Applications*, 37.4, pp. 3373-3379.

Kim, K. S. et al. (2011), Comparison of k-nearest neighbor, quadratic discriminant and linear discriminant analysis in classification of electromyogram signals based on the wrist-motion directions, *Current Applied Physics*, 11.3, pp. 740-745.

Kolari, J. et al. (2002), Predicting large US commercial bank failures, *Journal of Economics and Business*, 54.4, pp. 361-387.

Kuumar, P. R., Ravi, V. (2007), Bankruptcy prediction in banks and firms via statistical and intelligent techniques – A review, *European Journal of Operational Research*, Vol. 180, No. 1, pp. 1-28.

Laitinen, E. K., Lukason, O., Suvas, A. (2014), Behaviour of Financial Ratios in Firm Failure Process: An International Comparison, *International Journal of Finance and Accounting*, 3.2, pp. 122-131.

Lasisi, T. A., Shangodouin, D. K. (2014), *Robust Test for detecting Outliers in Periodic Processes using Modified Hampel's Statistic*.

Lee, S., Choi, W. S. (2013), A multi-industry bankruptcy prediction model using back-propagation neural network and multivariate discriminant analysis, *Expert Systems with Applications*, 40.8, pp. 2941-2946.

Odom, M. D., Sharda, R. (1990), A neural network model for bankruptcy prediction, *1990 IJCNN International Joint Conference on neural networks*, pp. 163-168.

Ohlson, J. A. (1980), Financial Ratios and the Probabilistic Prediction of Bankruptcy. Blackwell Publishing, *Journal of Accounting Research*, Vol. 18, No. 2, pp. 109-131.

Olson, D. L., Delen, D., Meng, Y. (2012), Comparative analysis of data mining methods for bankruptcy prediction, *Decision Support Systems*, 52.2, pp. 464-473.

Piotroski, J. D. (2000), Value investing: The use of historical financial statement information to separate winners from losers, *Journal of Accounting Research*, pp. 1-41.

Premachandra, I. M., Bhabra, G. S., Sueyoshi, T. (2009), DEA as a tool for bankruptcy assessment: A comparative study with logistic regression technique, *European Journal of Operational Research*, 193.2, pp. 412-424.

Psillaki, M., Daskalakis, N. (2009), Are the determinants of capital structure country or firm specific? *Small Business Economics*, 33.3, pp. 319-333.

Reisz, A. S., Perlich, C. (2007), A market-based framework for bankruptcy prediction, *Journal of Financial Stability*, 3.2, pp. 85-131.

Tabachnick, B. G.; Fidell, L. S. (2001), Using multivariate analysis, *California State University Northridge: Harper Collins College Publishers*.

Tinoco, M. H., Wilson, N. (2013), Financial distress and bankruptcy prediction among listed companies using accounting, market and macroeconomic variables, *International Review of Financial Analysis*, Vol. 30, pp. 394-419.

Wilson, R. L., Sharda, R. (1994), Bankruptcy prediction using neural networks, *Decision support systems*, 11.5, pp. 545-557.

Zanganeh, T., Rabiee, M., Zarei, M. (2011), Applying adaptive neuro-fuzzy model for bankruptcy prediction, *International Journal of Computer Applications*, 20.3, pp. 15-21.

Zavgren, C. V. (1985), Assessing the vulnerability to failure of American industrial firms: a logistic analysis, *Journal of Business Finance & Accounting*, 12.1, pp. 19-45.