ECONOMICS & Sociology

Shvedovsky, V., Standrik, A., Bilan, Y. (2016), Economic and Social Institutions: Modelling the Evolution Paths for the Archaic Society, *Economics and Sociology*, Vol. 9, No 2, pp. 137-147. **DOI: 10.14254/2071-789X.2016/9-2/9**

Vyacheslav Shvedovsky,
Lomonosov Moscow State University,
Moscow, Russian Federation,
E-mail: mamosp@mail.ru

Anton Standrik,
Lomonosov Moscow State University,
Moscow, Russian Federation,
E-mail: standrik@mail.ru

Yuriy Bilan,
University of Szczecin,
Szczecin, Poland,
E-mail: jurij@wneiz.pl

Received: January, 2016
1st Revision: March, 2016
Accepted: May, 2016

DOI: 10.14254/2071-789X.2016/9-2/9

ECONOMIC AND SOCIAL INSTITUTIONS: MODELLING THE EVOLUTION PATHS FOR THE ARCHAIC SOCIETY

ABSTRACT. Evolution of society is considered to be a result of development of social reproduction systems. The leading role in this process is played by the development of the creative and active parts of society where package discoveries and inventions form the basis of the dominant technology of production. Our paper presents a complex economic model of the system with the social reproduction as a set of reproduction kernels and a net of reproduction cycles. We use mapping of permutations to graphs for demonstrating that the evolution of society can be represented as a sequence of nested sub-groups in group lattices. Evolution paths and their complexity can be evaluated using the graph theory and the complexity criteria. Our findings show that these complexities correspond to those of the spectrum of cycles of the collective economy so that social constants can be calculated and used to describe transition between evolution phases from the Mesolithic to Neolithic age. This allows us to reconstruct a group-lattice of the evolution of society using mathematical methods.

JEL Classification: C02, C65, P51

Keywords: economic systems, social heredity, culture code, evolution, entropy, complex systems.

Introduction

There are two key elements that motivated the researchers to investigate socio-cultural differences between different cultures and ethnic groups: Firstly, every country and ethnos has its own national idea that is explicitly or implicitly stated (e.g. “wakon yosai” in Japan combining Japanese spirit and Western knowledge) (Sawamura, 2002). Secondly, every national idea is based on a great historical period of a specific country that preceded and caused the appearance of the idea. And history is a process of cultural codes’ mutation and accumulation (Canevacci, 1992). Cultural code is an object that represents cultural values as a sequence of characters. Therefore, there is an issue researchers have to face: using mathematical approach to describe the process of culture codes formation on macro-level (Malloy and Malloy, 1998). Cultural differences of Western and Eastern society models can be described by the means of sociology (Barkema *et al.*, 2015).

The most advanced from application potential point of view is the work of Rapaille (2007) that is dealing with the subject of studying various societies on the basis of "collective unconscious". Bourdieu (1990) employs the frequency analysis of texts used to study the subject that was also studied in Rapaille (2007) by the means of sociological polls and interviews. National idea in these terms can be described as a system of cultural codes that that for some time can be considered to be a constant source of society evolution component. Turchin (1977) is close to our approach from the point of complexity theory as in this work meta-system transitions are described and substantiated in the humanity evolution process. The idea of cognitive factor in evolution process of living matter that was also well-described in Wynn (2002, Donald (1993), or, most notably, Red'ko (2014), with the latest being complementary to our work. Also, we most certainly agree with Bourdieu (1990) that cultural code is something similar to every set of habitus (system of acquired dispositions functioning on the practical level as categories of perception and assessment or as classificatory principles as well as being the organizing principles of action) of a particular social system.

There are also other well-executed attempts to solve this problem that can be found in other research works (see e.g. Grinchenko and Shchapova, 2010; Lakic and Draskovic, 2015). However, their attempt is considered to be questionable because of the lack of substantiation for the statement that periods of human society development can be matched to a range of Fibonacci numbers. Other approaches that are described in other publication (see e.g. Walton and McKersie, 1965) are much closer to our approach from the point of methodology. However, some authors describe the methods that use labor division as a main criterion and therefore are more detailed and do not cover the very early phase of society evolution when gathering and hunting was the main domain of the human development. From our point of view, such approaches also provides questionable results by applying cultural values table to the global history without mentioning how these values were formed. From our point of view it might be much more correct to consider mega-historical role of cultural codes in lifestyles of nations and ethnos as it is done in Rapaille (2007). The effectiveness and high quality of Japanese labor is assumed to be a consequence of tough living conditions of high-populated nation with scarce natural resources. Antithetic situation was observed in America where the opposite cultural code hindered effective application of Japanese business strategies (see Rapaille, 2007, p. 108).

1. Research methodology

In outlining our research methods, we make an attempt to overcome the difficulties described in the similar works of other authors. These works are based on an idea that an increasing difficulty of social relations can be represented by increasing number of social institutions (kernels) and relations (evolution cycles) between them. This idea is the basis of society evolution model that is going to be described. Therefore, our main objectives in this paper are the following:

- Describing group-lattice approach to the modeling of evolution of archaic society;
- Presenting the law of changes in quantitative and qualitative forms;
- Showing the possibility of mathematical description (by the means of graph theory and algebra) of the differences in culture code formation that were caused by the differences of starting conditions and evolution paths of nations and ethnic groups, the differences between Eastern and Western cultures.

The most important task, however, is to study opportunities of the new cultural codes appearance and development in order to overcome world motivation crisis.

As it was mentioned before, the main theoretical instrument of the approach is graph theory and knowledge about symmetric groups. Graph vertices are associated with social institutions (reproduction kernels). Social reproduction cycles can be represented by permutations and mapped to directed graphs.

With regard to the above, we can define the reproduction kernel – a system of living elements that has input and output and converts some elements to other. It has at least three components:

- Conversion technology;
- Living force (for instance bacteria, animals, humans, etc.);
- "Capital" – The existing system of material, energy and informational objects that are connected by a certain infrastructure. For example for organic processes "capital" consists of biotopes and some living creatures while in economy "capital" consists of assets, funds, manpower, etc.

2. The hypotheses

Therefore, a reproduction cycle that can be defined represents an integral functional system of ordered in time and space processes. Example of Reproduction cycle is the Marx's Metaphor (see e.g. Carver, 1998).

Social reproduction system represents a set of reproduction kernels covered by the range of reproduction cycles. First of all, reproduction cycle is considered to be one of four main types: "life of people", "life of things", ecological cycle, and cognitive cycle. Then, digraph's arcs are associated with main economic processes: p (vertical arc, top down) – production, d (horizontal arc) – distribution, e (two horizontal arcs) – exchange, c (vertical arc, bottom up) – accumulation, u (loop) – utilization. All these types of relations are assumed to be in a system where the structure of the economy and society remains the same.

Thence, we can articulate our *first research hypothesis*: Reproduction cycles can be mapped to digraphs and be represented by permutations. The initial graph corresponds to the starting phase of primitive society development where neighboring clans lived and used their biotopes as food source. That graph has 4 vertices. Permutation group on complete graph is non-Abelian symmetric group S_4.

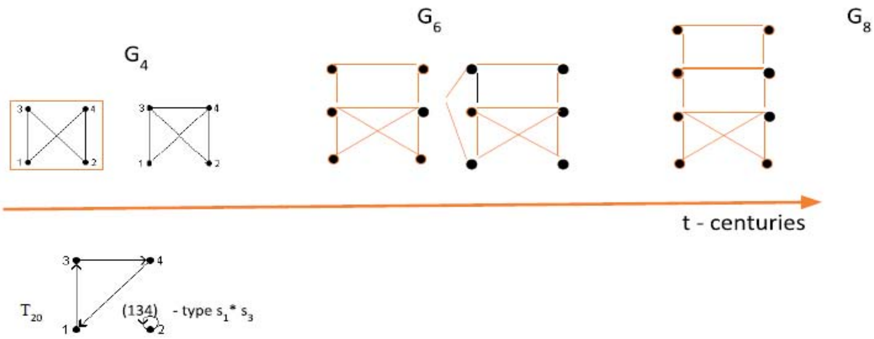
Moreover, we can come up with *the second research hypothesis*: social development level corresponds to its complexity level that is estimated by a spectrum of reproduction cycles in a system that is represented by a subgroup of symmetric group. Transition from simple to complex systems is carried out by making discoveries and inventions and the complexity of these inventions corresponds to the complexity of society development level – that's the idea of the Law of Changes (see Bekhterev, 1993). The approach based on the following sociological postulates:

- Formed lifestyle as a sustainable spectrum of reproduction cycles that is taking place during a long period of time determines mindset and set of cultural values of some society.
- Mindset generates perception filter.
- Peculiarities of various perception filters allow members of various societies to find unique features of this world (for example Chinese doctors can distinguish between more than a dozen different types of pulse and Eskimo know 10 at least 10 types of snow).
- This filter together with knowledge base allows society to make inventions and to progress to a new complexity level.
- In these terms there are two types of society development: firstly society develops as the number of reproduction cycles and the order of subgroup of symmetric group increase; secondly, as the order of the subgroup increases a breakthrough that is characterized by increase of graph vertices (that symbolize social institutions) number,

can happen. Every breakthrough means increase in social memory capacity and thereby means increase in social complexity. The appearance of new institution can be marked by special archeological events or markers. For example: appearance of “the Elders” institution is marked by the appearance of graves of high status tribe members; appearance of initiation institute can be marked by petroglyphs that pictured the process of hunting and were used as a memory storage or knowledge base.

3. The model and key results

The early discoveries are stored as a social heritage and remain in perception filter and reproduction system. This process of memory storage and accumulation is implemented by nested chains of subgroups that represent the process of society evolution. The evolution process begins as a set of reproduction cycles on graph with 4 vertices – i.e. as a subgroup of symmetric group S_4. As more reproduction cycles appear and nest in each other– the probability of new social institution appearance grows and then more and more vertices become involved in society lifestyle and more complex relations take place. Since S_4 (with trivial two last places) is subgroup of S_6 and in the same way S_6 is subgroup of S_8 the evolution of Social reproduction system can be represented by sequences of nested permutation groups where permutations are mapped to graphs (see *Figure 1*).



1	2	3	4
4	2	1	3

Figure 1. Evolution graphs representing social reproduction systems over time
Source: own results.

$$\ldots \subset Hn \subset Hm \subset Hp \subset \ldots,\ n < m < p \qquad (1)$$

where Hn, Hm, Hp – are subgroups of symmetric groups with n, m and p elements correspondingly.

There are adjustments that need to be made to understand the whole picture: firstly, social modeling is limited to logics of inner development, i.e. it does not take into account external forces and effects (for example Crete volcano eruption that caused the destruction of Minoan Civilization). Secondly, different number of vertices marks different social structure types at different phases of evolution: clan communities with patriarch (food gathering economics); tribe with Council of Elders (bow-hunting period); tribal alliances with chieftain (the beginning of military conflicts for food resources).

Every evolution phase is finished by a subgroup in group S_8 and then it's assumed to be wrapped and every new evolution phase begins with S_4 where every vertex has knowledge about previous evolutionary process. This knowledge is stored in its cultural code. For example, in the first phase loop in vortex "society" means anthropophagy but in the second phase (agricultural economics), the loop in vertex "society" means usage of old means of food production (hunting and gathering if it is accessible. If it is not, then anthropophagy happens. It means that transition to new level allows using technologies from previous levels that are stored in social memory in form of cultural code. Graph S_4 and S_6 in these systems are believed to be complete as it is necessary for systems in model "WE – THEY" to be connected and it is impossible for graph for permutation to be incomplete if the initial graph for permutation has only one connection component (see the Appendix).

The complexity of innovations package that is being mastered is proportional to the complexity level of reproduction cycles. The quantitative representation of this law that is based on stated postulates can be represented as:

$$|S_n| = n!\ H(n) = C + \ln n!$$
$$H(n) \approx \ln L + (n-1)\ln n - \ln \ln (n-1) - n \qquad (2)$$

n – rank of group Sn for Social reproduction system. It denotes number of reproduction kernels. Therefore, maximum number of reproduction cycles in reproduction system is n!; L – social constant that can be evaluated with the usage of archeological and sociological data (Shvedovskiy, 2013). H(n) – system complexity. Size of permutation multiplication table (Cayley table) is n!*n!

All permutation types can be divided into topologically similar graphs that are determined in Poe's law (see Harary and Palmer, 2014). For instance, the Poe's law for enumeration of group S_4 orgraphs with 5 permutation types from the cyclic indices of group S_4 can be presented as follows:

$$Z(S_4)=1/24((s_1)^4 + 6*(s_1)^2 *s_2 +8*s_1*s_3 + 3*(s_2)^2+ 6*s_4) \qquad (3)$$

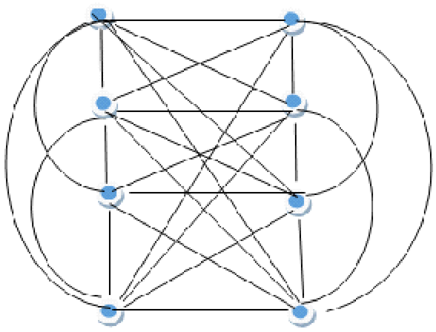


Figure 2. Complete eight-vertex graph
Source: research of the authors.

Integer coefficients of the polynomial define the number of orgraphs of certain type, for instance, 6 for $(s1)^2$ *s2, i.e. determine their weights in the whole spectrum of all possible orgraphs (that represent reproduction cycles). Also it has to be taken to account that it is not necessary for the graph to be complete:

$$Z(S_6) = 1/6!\ (s_1^{\,6}+15\, s_1^{\,4} s_2+ 40\, s_1^3\, s_3+45s_1^{\,2}\, s_2^{\,2}+ 90s_1^{\,2}\, s_4+ 120\, s_1\, s_2\, s_3 + 144s_1\, s_5 + 15s_2^{\,3} + 90\, s_2\, s_4+ 40\, s_3^{\,2}+120\, s_6) \qquad (4)$$

$$Z(S_8)=1/8!\,(s_1^8+28\, s_1^6 s_2+112\, s_1^5 s_3+ 210\, s_1^4 s_2^2+ 420\, s_1^4 s_4+ 1120\, s_1^3 s_2 s_3+1344 s_1^3 s_5+420 s_1^2 s_2^3+2520\, s_1^2 s_2 s_4 +1120 s_1^2 s_3^2 +3360 s_1^2 s_6+1680\, s_1 s_2^2 s_3 +4032\, s_1 s_2 s_5 + 3360\, s_1 s_3 s_4 +5760\, s_1 s_7+105\, s_2^4+1260\, s_2^2 s_4+1120\, s_2 s_3^2+3360\, s_2 s_6+2688\, s_3 s_5+1260\, s_4^2+5040 s_8) \qquad (5)$$

Here, the bottom two vertices in *Figure 2* represent cultivated areas that are used as source of food for example by tribal unions.

4. Main implications: economic and social institutions

From the above, one can see that very new pair of vertices means the emergence of new social institution. The emergence of new social institutions means increase of capacity and capabilities of social memory.

The first improvement of social memory was made by creation of institution of “the Elders” which is represented on graph G_6 by top two vertices. The Elders distributed food supplies among the tribe members and did their best to prevent consanguineous marriages. Archeological merger of this is represented by the high status men graves.

The second improvement of social memory involves the usage of stone and organic items for storing some information. For instance beads of different colors and shapes could be used to differ one tribe from others and seashells were used to remember debts. Cave drawings were used to mark clans’ living areas and to store information about hunting methods – this is represented by top two vertices in graph *Figure 2*.

We employ the method of constant L calculation for every historical phase. It is important that periodical usage of permutations on graphs with 4, 6 and 8 vertices does not mean strict repeating of mathematical forms. At the second phase of human evolution when clans are united in tribe communities – model starts with graph of 6 vertices, it means that tribe communities have elders’ council and nomadic lifestyle is limited to some specific area while other areas belong to other tribes and tribe communities. Population growth leads to the next evolution phase where tribe communities are united into tribe alliances and ruled by chieftains as struggle for resources becomes tougher. This phase is a transition phase from gathering economics to sedentary and it’s the point where differences between Eastern and Western social models become observable and that’s the point where nations chose different evolution routes that were going to influence cultural values. However there are differences in group approach in modeling physical processes and processes in living matter: symmetric groups that are used to describe economic system are non-Abelian unlike Abelian groups that are used in physics. Phase of gathering economy is characterized by complete symmetric groups on complete graphs. Running all appropriate calculations, one can see that all evolution paths from group S_4 to group S_8 where the evolution begins in node “0” and ends in “701”.

Transition to producing economy happens if and only if society has developed instruments of social memory such as cave paintings and a number of social institutes: institute of initiation as a system of knowledge transfer, institute of elders who distribute food resources and prevent intermarriages. Also, there is an institute of ‘primal money’ with a function of remembering debts – neighboring tribes help each other and use some tokens to remember debts.

Because of differences in starting conditions that became significant after transition to sedentary farming, graphs that represent relations between biotopes and communities in Western and Eastern models are different.

If the ratio of tillage to pasture (1:3) cannot be met, in such a situation biotopes cannot fulfill all needs as a food source so that food have to be gathered by the meanings of old methods: gathering and hunting. It means that links with bottom nodes are weak and can't be taken into account. Considering structure development in this society, two types of society structures need to be observed.

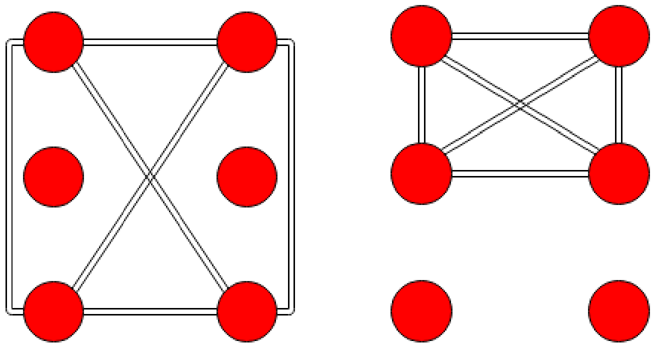


Figure 3. Differences between evolution graphs of Western (left) and Eastern (right) models
Source: Own results.

Interpretation of the *Figure 3* can be done in the following way:

- Communities try to survive together and help each other by sharing food and living force.
- Elite is using land to the bitter end while other households are surviving (or not surviving) by themselves.

In these situations the role of culture codes is crucial that strengthen or weaken evolutionary potential. Examples how these codes helped or hinder societies to survive: a) restriction of consuming fish led to destruction of Vikings colony in Greenland, or b) restriction of consuming pork reduced number of diseases among Muslims.

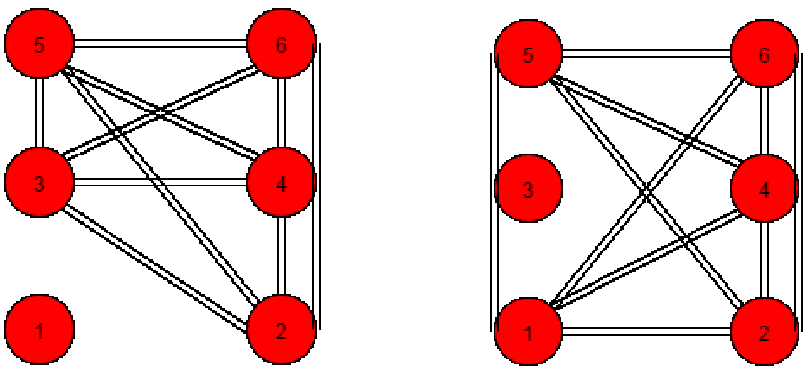


Figure 4. Graphs with one food vertex (left) and one society vertex (right)
Source: Own results.

Calculations on two sets of groups that can be implemented with two different graphs (*Figure 4*) show that for the first graph there are 1372 arcs that are responsible for social communications while for the second graph there are 732 arcs. For the largest group (final node in corresponding group lattice) for the first graph there are 192 communication arcs while for the second graph there are 96 arcs that represent communications between neighboring social units in model "WE – THEY". It means that lack of food and natural resources stimulates communities to unite in order to survive, leads to early governmental formation and creates a basis for the appearance of traditionalistic cultural values.

If one was to represent these results in a form of a table and use normalization, the differences in rooting indexes between Western and Eastern models would look like the following (*Tab. 1*).

Tab. 1. Differences in rooting indexes between Western and Eastern models

Value / Location	EAST	WEST	EAST(normalized)	WEST(normalized)	Δ(%)
Collective responsibility	10702	9645	1.00	0.90	10
Respect for the hierarchy	13069	12873	1.00	0.98	2

Source: Own results.

Using these terms, we can suggest that in considered historical phase there was almost equal respect for the hierarchy in both models while culture values of collectivism in East model already started to emerge.

It means that with the same complexities graphs can represent different social behavior that depends on cultural codes that are represented by groups of social reproduction cycles. In Russia, until the 1960s and their cultural values, that are believed to belong to "Eastern" lifestyle, were dominant as one of the main economic directions was agriculture. In his work Rapaille (see Rapaille, 2007, p. 108) gives a convincing example that a superficial desire of the Americans to use Japanese strategies to increase production quality, but without considering differences of cultural codes resulted in a failure. The generalization of this idea might help to understand a hypothesis about liberal reforms failure in Russia.

Conclusions

Overall, our results suggest that macroevolution of society can be represented with chains of nested subgroups of symmetric groups. Unlike processes in physics, groups in social processes are non-abelian.

Historical and archaeological data allows us to create sequences of nested subgroups that represent the process of society evolution, and finally to create an evolution group lattice. However, one can see that societies in the phase of gathering economics did not have differences in social structure. This approach gives us an opportunity to evaluate the rooting depth of certain cultural codes in the context of evolution of certain society.

Cultural codes and their rooting depths' indexes can be evaluated and interpreted in terms of individualism or collectivism mindset domination in a certain society. One can suggest that the possible cause of failure of numerous liberal reforms in Russia (3 times in the 20th century alone) was the incompatibility of liberal cultural codes with cultural codes of Russian traditional lifestyle.

Consideration of the history and all its failures leads to the idea that world motivation crisis can be overruled by acceptance of and respect paid to the system of cultural codes of each society that has long history and a path formation. These cultural codes, if used properly, can enhance the economic and social development of the countries in question.

References

Barkema, H. G., Chen, X. P., George, G., Luo, Y., & Tsui, A. S. (2015), West meets East: New concepts and theories, *Academy of Management Journal*, Vol. 58, No. 2, pp. 460-479.

Bekhterev, V. M. (1993), *VM Bekhterev's Collective Reflexology*, Nova Science Pub Incorporated.

Bourdieu, P. (1990), *The Logic of Practice*, Cambridge: Polity.

Canevacci, M. (1992), Image accumulation and cultural syncretism, *Theory, Culture & Society*, Vol. 9, No. 3, pp. 95-110.

Carver, T. (1998), *The Postmodern Marx*, Manchester: Manchester University Press.

Donald, M. (1993), Human cognitive evolution: What we were, what we are becoming, *Social Research*, Vol. 60, No. 1, pp. 143-170.

Grinchenko, S. N., & Shchapova, Y. L. (2010), Human history periodization models, *Herald of the Russian Academy of Sciences*, Vol. 80, No. 6, pp. 498-506.

Harary, F., Palmer, E. M. (2014), *Graphical enumeration*, Elsevier.

Lakic, S., Draskovic, M. (2015), Implications of Institutional Dispositions of Neoliberalism, *Montenegrin Journal of Economics*, 11(2): pp. 113-124.

Malloy, C. E., & Malloy, W. W. (1998), Issues of culture in mathematics teaching and learning, *The Urban Review*, Vol. 30, No. 3, pp. 245-257.

Rapaille, C. (2007), *The culture code: An ingenious way to understand why people around the world live and buy as they do*, Crown Publishing.

Red'ko, V. (2014), Modeling of cognitive evolution: View from Artificial Intelligence, *Logos*, Vol. 97, No. 1, pp. 109-140.

Sawamura, N. (2002), Local Spirit, Global Knowledge: a Japanese approach to knowledge development in international cooperation, *Compare*, Vol. 32, No. 3, pp. 339-348.

Shvedovskiy V.A. O vychislenii social'nyh konstant v modelirovanii evolyucii arhaichnogo sociuma // Prostranstvo i vremya, 4(14), M.: 2013, pp 72-80.

Turchin, V. F (1977), *The Phenomenon of Science: A Cybernetic Approach Human Evolution*, New York: Columbia University Press.

Walton, R. E., McKersie, R. B. (1965), *A behavioral theory of labor negotiations: An analysis of a social interaction system*, Cornell University Press.

Wynn, T. (2002), Archaeology and cognitive evolution, *Behavioral and Brain Sciences*, Vol. 25, No. 3, pp. 389-402.

Appendix

Permutation graph for permutation $A = (\sigma_1, \sigma_2, ..., \sigma_n)$ of order n, where $\{\sigma_1, \sigma_2, ..., \sigma_n\}$ – set of integer numbers from 1 to n, is a bijection of permutation A to directed graph $G = \{X, Y\}$, where X – set of vertices of order n and Y – set of graph's arcs, and every vertex from X has one to one mapping to a corresponding integer position α_i in permutation A. Set Y consists of pairs (α_i, σ_i) where element σ_i is placed at index α_i in permutation A;

Underlying graph for subset w of a symmetric group S_n is a surjective mapping of w to non-oriented graph $V= \{M, N\}$ where M – set of graph vertices of order n where every vertex from M is mapped to α_i – place number(index) in permutations from S_n, and N – set of graph edges where an edge $\{\alpha_i, \sigma_i\}$ belongs to N if and only if either (α_i, σ_i) or (σ_i, α_i) is present at least in one permutation in set w;

Closure of subset K in group A is a subgroup of minimum size in A that contains K.

The process of subset in group extension: given a subset $M = \{a_1, a_2, ..., a_m\}$ of finite multiplicative group G. The process of subset M extension to group in G is a process that acts according to the algorithm:

- B is a set of results of all possible multiplications of elements in M.
- B is combined with M: $M = M \cup B$.
- If B is not empty then B:= ∅ and algorithm returns to (a.) where M is a new set from (b.). If B = ∅ then algorithm stops with output M.

Lemma: If M is a subset of symmetric group S_n then applying the process of extension to M in S_n makes a Closure K in S_n.

Proof: is a consequence of subgroup criterion: nonempty subset H of group G is a subgroup of G if and only if:

1. H – finite set;
2. $h1 * h2 \in H, \forall h1, h2 \in H$.

First condition is satisfied because S_n is a finite group and subset of a finite group is a finite set. Second condition is satisfied as a direct consequence of the process where elements are multiplied to each other. This subgroup is minimal because any other subgroup that contains set M also contains K due to the algorithm (if it doesn't – some multiplication results won't be in this subgroup thereby it won't be subgroup).

Theorem: Here M is a subset of symmetric group S_n. V – underlying graph of M. Application of The Process to M makes a Closure in S_n and V is transformed into a corresponding underlying graph that consists of k complete graphs where k is a number of connected components in a transformed V.

Proof: Firstly we need to prove that permutations' set that corresponds to connected but non complete graph can be closured (by application of the process) to a set that corresponds to complete graph. It's sufficient to prove that in closure for every pair of vertices that has a path between them there is path of length 1 i.e. there is an edge between corresponding vertices in Underlying Graph. Let's assume that there are vertices α and β, and there is path between them that consists of vertices $\gamma_1, \gamma_2, \gamma_3, ..., \gamma_m$. Presence of this path between α and β means that in corresponding permutation set M there are elements: γ_1 that is placed in position α, γ_2 that is placed in position γ_1, γ_3 that is placed in position γ_2, …, β that is placed in position γ_m.

With regard to these terms and taking into account that closure is a group and for every element there is an inverse one, we can state that if there is a permutation where ε is in position κ then there is a permutation where κ is in position ε.

Permutation multiplication operation is superposition. Therefore, multiplication of permutation a where κ is in position ε by permutation b where γ is in position κ, gives permutation c where γ is in position ε.

Following this logic and considering path α, γ_1, γ_2,...,γ_m,β, we see that multiplication of permutation where γ_1 is in position α by any other permutation(including self) where γ_2 is in position γ_1, gives permutation where γ_2 is in position α. We obtain an edge (α, γ_2) that did not present in the initial path. Adding new permutation to the initial set and using this method again we get edges (α, γ_2), (α, γ_3),... and, finally, (α, β).

Different connection components during the process do not intersect because there were no edges or paths between them in the initial set, and they do not appear as the connection appears, if and only if there is a path in underlying graph from one vertex to another.